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LETTER TO THE EDITOR

Discussion of "Strain hardening in the moving hinge method", *Int. J. Solids Structures*, Vol. 30, pp. 3475–3489 (1993).

The paper discusses the nature of discontinuities in structures made of a rigid–strain hardening material and works out a detailed example. This is a subject of great practical significance. Can simplicity of rigid–perfectly plastic solutions be extended to a rigid–work hardening material? The paper contributes to resolving this question.

The difficulty in formulating the problem is that at the propagating discontinuity the stress state is not uniquely determined. The rate of plastic work at the discontinuous velocity field, using the authors' notation is

$$\dot{E}_{\mathrm{H}} = M[\dot{\theta}] = M\dot{H}[K]. \tag{1}$$

In a rigid–perfectly plastic material $M = M_{\mathrm{p}}$ but in the case of a rigid–work hardening material the bending moment within an idealized hinge cannot be found from the constitutive equations alone. An additional assumption is needed here. I think that this point is not recognized by the authors. The presence of strain hardening is smearing-out the hinge into a finite region. Therefore, in reality, there is a one-to-one correspondence between the curvature and the bending moment (Croll, 1985). When the width of this finite region shrinks to zero, the bending moment in the hinge is undefined, bounded by

$$M(K^-) < M < M(K^+). \tag{2}$$

In Ref. 19 it was additionally assumed that

$$M = M(K^+). \tag{3}$$

The present authors postulated that there is a linear variation of the bending moment with curvature within the hinge. This is equivalent to saying that the hinge bending moment is an arithmetic average

$$M = 1/2[M(K^-) + M(K^+)]. \tag{4}$$

The authors are reluctant to admit that (4) is an assumption. A justification of the above statement is given below.

The cross-sectional bending moment, per unit width, is defined by

$$M = \int_{-h/2}^{h/2} \sigma(\varepsilon) z \, \mathrm{d}x. \tag{5}$$

In a linear strain hardening material the stress is related to the strain by

$$\sigma = \sigma_{\mathrm{y}} + E_{\mathrm{p}}\varepsilon. \tag{6}$$

The strain is, in turn, linearly related to the curvature change, according to the Love–Kirchhoff hypothesis:

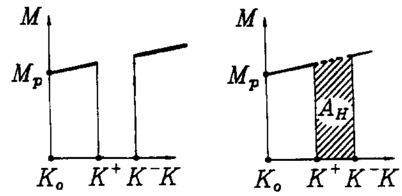

Fig. 1. Non-uniqueness of a bending moment within a hinge and a linear approximation.

$$\varepsilon = z(K_s - K_0). \tag{7}$$

Using eqns (5)–(7), eqn (16) of the original paper is derived.

Note that eqns (5) and (6) are valid everywhere, including the cross-section where the hinge appears. By contrast, eqn (7) is true only if the curvature K_s is uniquely defined. In the plastic hinge, the curvature is not uniquely defined and neither is the bending moment. Therefore, Figs 4(b) and 4(c) of the original paper should be modified by showing a gap on the moment–curvature diagram. It becomes clear that one has to make an additional assumption to render the problem unique. Either a continuous change of curvature within the width of a hinge is specified or the variation of bending moment within the interval (K^+, K^-) is assumed. The simplest function is a linear one (broken line in Fig. 1) and this assumption was made by the authors.

PROFESSOR T. WIERZBICKI
Department of Ocean Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139, U.S.A.

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